

Assessing Status of Student's Conception about the Sequence of Numbers

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Abstract

The purpose of this study is to assessing status of student conceptions that are not in accordance with the formal concept. Data were collected from 37 students by assigning them to a sequence of numbers task and interviewing them. The concepts used by the students were recorded and clinical interviews were conducted to discover what motivated the students to adopt new concepts or caused them to resist new concepts. A theory of status conception was used as a theoretical framework for interpreting the results, which indicates that a variety of factors influence student resistance to new concepts. The findings of this study showed the status of student conceptions about the sequence numbers that were classified as being the status of intelligible but not both plausible and fruitful. Student conceptions of the sequence of numbers are: (1) students interpret that sequence must have a pattern, (2) students represent sequence numbers simply as arithmetic or geometric sequences, (3) students could not show that the sequence of numbers is a function and (4) in solving the problem of the sequence of numbers, students interpret terms of the pattern as the difference between the values of the sequence of numbers.

Keywords: Assessing, Conceptions, Sequence of numbers, Fruitful, Intelligible, Plausible

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I. Introduction

Before learning in the classroom, students are not merely a "tabula rasa" (Pinker 2003; Resnick, 1983; Chi & Roscoe, 2002). In fact, they have had ideas about an object and its properties based on their personal experiences from their previous education, from their interactions with parents and friends, and also from their reading books or other media. The ideas of individual students about the meaning of a concept that are subjective are called as conception. Sfard (1991) defines conception as an internal image that belongs to students about a concept. Likewise, Glynn & Money (1995) state that conception is students' internal image as a result of external image from entities built by others such as teachers, textbook writers, or software designers. Leatham (2006) refers to conceptions as conscious or subconscious beliefs, understanding, meaning, mental images, and preferences.

In general, students' conception can be classified into two. The first one is students' conception which is in accordance with scientific concept. Such conception is called as a scientific concept (Vosniadou & Brewer, 1994); expert concept (diSessa, 2002); formal concept (Tall, 2008). The second classification is students' conception which contradicts with expert concept/scientific concept. In several studies, used some different terms such as *preconceptions* (Chi & Roscoe, 2002), *alternative conceptions* (Swan, 2001), *misconceptions* (Durkin & Rittle-Johnson, 2015), *alternative frameworks* (Nussbaum & Novick, 1982), *naive knowledge* (diSessa, 2002).

In mathematics and science instruction, conception is commonly identified among students (Reiner, Slotta, Chi, & Resnick, 2000). Kesan & Kaya (2007) stated that the reason for that matter is that students have difficulties in understanding scientific concepts. An example of conception in real analysis instruction is found in real number sequence material (called henceforth as sequence). At senior high school, sequence is introduced as collection of numbers sequenced according to a certain pattern, such as arithmetic and geometric sequences. Sequence and series are usually discussed in an integrated unit. In higher education, sequence is seen from an analysis perspective and considered as a specific form of a function. Sequence numbers is a function defined on the set \mathbb{N} of natural numbers whose range is contained in the set \mathbb{R} of real numbers (Bartle & Sherbert, 2011). Definitions of sequence are commonly presented in a formal shape with mathematics symbols. According to Tall (2008), such definition is labeled as formal definition.

Some research findings showed the importance of conception in which it is considered as one of basic factors to learn next concepts. However, students' conception own is often incompatible with scientific concept

and inhibits them to understand scientific principles and concepts (Treagust, 2008; Chi & Roscoe, 2002). In this case, students have to be able to evaluate their incorrect conception and replace it with new and more scientific conception. Students' conception of different with the scientific concept enables conceptual changes will occur already has. Conception of students as important in conceptual change. Conceptual change involves the interaction between the new knowledge and the existing knowledge in order for the new knowledge to be reconciled with the old ones (Hewson et al., 1998). Posner et al. (1982) argued that conceptual change is a process by which a learner's existing conceptions may be replaced by more robust ideas.

This study is aimed at assessing students' conception about sequence of number based on Chen and Wang (2008) idea of conception status. According to Hewson (1981), conception status is the extent students know, receive, and discover a useful idea. Intelligible status of students' conception can be seen from the way they explain meaning, give examples and non-examples, and represent their conception in other forms. Plausible status of students' conception can be identified from the way students give rational and scientific reasons of their conceptions and relate their conception with other concepts. Finally, fruitful status of students' conception can be traced from how students implement their conception to solve other problems.

Status of students' conception is very useful to evaluate changes of students' conception during their learning (Hewson & Thorley, 1989; Hewson & Lemberger, 2000; Treagust & Duit, 2008). Therefore, exploring status of students' conception will provide description of students' basic knowledge of sequence, give contribution for lecturers to do conceptual changes and plan for more appropriate instruction.

II. Method

This study is qualitative in nature involving 37 of third-semester prospective teacher students of the Mathematics Education Department. The data were collected through two stages of a worksheet and interview. In the worksheet, students were required to solve basic problems with the concept of sequence of numbers. Two questions were employed in the worksheet; the first question has the purpose of knowing student conceptions of sequence definition, while the second one is to know students' conception of the n-th term from a sequence of numbers. The questions are as follows:

1. What do you know about the sequence of numbers concept?
2. Given sequence 2, 5, 8, ...
 - a. Determine the fourth term of the sequence. Give your reason!
 - b. Adi determines that the fourth term of that sequence is 17. Give your reason to Adi's opinion!

The students were grouped into 2 criteria: (1) conception which contradicts formal concepts, and (2) conception which is in accordance with formal concepts. The steps in grouping the subjects are: (1) giving sequence of numbers task, (2) promoting interview to confirm students' answers in written and to obtain uncollected information from the written test, and (3) grouping students into two criteria: a conception which contradicts with formal concepts, and conception which contradicts with formal concepts. Furthermore, student concepts that are not in accordance with formal concepts will be categorized openly in response to student responses.

The criteria used to assessing status of students' conceptions about the sequence of numbers are as the following.

Table 1. Criteria for status of student conceptions.

Condition	Criteria
Intelligibility	<ul style="list-style-type: none"> • meaning clarity and conception meaning • example and non-example of conception
Plausibility	<ul style="list-style-type: none"> • Conception representation in the forms of images/ illustration, symbol, or sentences • rationality and scientific degree of conception.
Fruitfulness	<ul style="list-style-type: none"> • relatedness of the conception with other concepts. • ability of the conception to solve the problems.

Some factors related to conception status were identified from the interview transcripts to help the process of classifying students' conception about sequence of numbers into intelligible, plausible, or fruitful. Besides, an interview was needed to reveal further about possible sources of students' conception.

III. Result

Here is the explanation of the research's results concerning with students' conceptions about sequence of numbers. There were 37 students consisted of two criteria: 25 conception which contradicts with formal concepts and 12 conception which accordance with formal concepts.

Table 2. Students' Conception of Sequence Numbers

	Categorizing of students' conception	Percentage
1.	conception which accordance with formal concepts	32,43
2.	conception which contradicts with formal concepts	67,57

Students whose conceptions do not match the formal concept, they are divided into 2 categories. A total of 17 student conceptions are included in the category that a sequence is a collection of numbers that has a pattern. The 8 students of conception belong to the category that the sequence is a collection of numbers that have the same difference.

Dealing with students' responses to question 2a which required students to determine the fourth term of sequence 2, 5, 8, ..., all students answered that the fourth term is 11. To arrive at the answer of the fourth term of the sequence, students used nth term formula of an arithmetic sequence, that is $U_n = a + (n - 1) b$.

Twenty-nine students showed their disagreement that the fourth value of sequence 2, 5, 8, ... is 17 (question 2b). They argued that since sequence 2, 5, 8, ... is arithmetic sequences with the first term of 2 and the difference of 3, they could determine by using nth term formula that the fourth term is 11, not 17. On the other hand, eight students agreed that the fourth term of the sequence is 2, 5, 8, ... is 17. They had various arguments. For example, students said that it is possible that Adi uses different formulas, not the n-th term formula of the arithmetic sequence.

In addition, based on the results of the interview with a student, it was gotten that there are some sources of student conceptions. One of them is students' previous knowledge they got from their senior high school. During their learning in senior high school, students learned a sequence starting from the definition of number pattern as shown in the following interview transcripts.

P : When do you know the sequence for the first time?

M : In senior high school

P : What concept base sequence?

M : number pattern

P : What is meant by number patterns?

M : There's a formula for it.

Besides, students' conception of the sequence was gotten from their teacher's explanation during their learning in the classroom.

There are 25 out of 37 students who have student conceptions that are not in accordance with formal concepts, 17 students have included in the category that a sequence is a collection of numbers that have a pattern, 8 students belonging to the category that the sequence is a collection of numbers that have the same difference. M1 is a student who has the conception that a sequence is a collection of numbers with a pattern. M2 is a student who has the conception that a sequence is a collection of numbers that have the same difference. Next, the status of students' conception in a sequence is revealed based on the classification of the conception status, they are intelligible, plausible, or fruitful. The status of student conceptions from each subject can be described as follows.

3.1. Conception Status of Subject M1.

Base on the results of subject M1's work on written test, it can be known that subject M1's conception on definition of sequence is that it is order of numbers in rows having a certain pattern among their term. While M1's answer on question 2a requiring to determine the fourth value of sequence 2, 5, 8, ... is 11. He used arithmetic sequence formula to solve that problem. However, M1's response to question 2b showed his agreement with the statement that the fourth term of sequence 2, 5, 8, ... is 17. The reason is that the known sequence is not necessarily arithmetic sequences, and it is possible that there is other different pattern. In order to explore M1's status of conception in great detail, the following explanation is presented.

Intelligible

To reveal status of students' conception on sequence more clearly, the subjects were required to explain the meaning of their conception, give example and non-example of their conception, and represent the meaning of their conception in other forms. The following interview transcript shows the meaning of definition of number sequence given by subject M1.

P : Can you explain once more about number sequence?

M1 : To me, sequence is order of numbers in rows with a certain pattern in which the term are connected with each other

P : What is the key words for your definition?

M1 : Order and pattern

P : What does it mean?

M1 : When we write sequence, such as 2, 3, 4, 5, ... but we write them this way 3, 2, 4, 5, So, the pattern is different

From the result of interview, it can be identified that there is no difference between M1's conception with his work in the written test. In addition, the meaning of sequence meaning is order of numbers by which pattern of sequence can be determined.

In giving example of sequence, M1 always relates a sequence with a certain pattern, so he cannot provide non-example of number sequence from his conception. The following quoted transcript of interview shows status of M1's conception when giving example of number sequence from his/her conception.

P : Can you give other example of sequence?

M1 : +1, -2, +3, -4, +5, -6,

P : Why do you think that it is an example of sequence?

M1 : There is a certain pattern, the pattern is changing from positive to negative and so on

P : Can you give an example of number order which does not have a pattern?

M1 : No, every number order must have a pattern

M1 gave an example of sequence with changing sign. His reason in giving such example of +1, -2, +3, -4, +5, -6, ... is because that sequence has a pattern, that is changing the sign from positive to negative. Then, he said that every number order must have a certain pattern. This shows that status of M1's conception is only being able to provide example from his conception. To further reveal his intelligible status, the interview was continued as follows.

P : What is the definition of sequence if it is presented in other form?

M1 : Do you mean I should write it mathematically, Sir?

P : Okay

M1 : Formula for arithmetic sequence is $U_n = u_1 + (n-1)b$

P : Is there any other sequence?

M1 : Yes, as I know, there is geometric sequence

P : Is the definition of sequence different when it is in represented in the form of symbol?

M1 : Yes, Sir, if the characteristics of arithmetic sequence is its values have the same differences

P : How is the general form of sequence?

M1 : How? I'm confused, Sir.

Based on the above interview, M1 could only represent his conception of sequence symbolically. He could not represent it in general; number sequence was only represented separately. For example, he represented arithmetic sequence with $U_n = u_1 + (n-1)b$. M1's conception on kinds of sequences was only limited on arithmetic and geometric sequences.

Plausible

In order to explore status of students' conception which is related with plausible status, subject's responses to question is considered. Quotation of the following interview shows how M1 responded question 2.

P : For question 2, sequence 2, 5, 8, ... is given. What is the fourth value?

M1 : 11, Sir

P : Why 11?

M1 : According to my interpretation, the sequence is arithmetic one because there is the same difference of 3. However, it is possible that the pattern is not like that when the test developer doesn't want it that way

P : What do you mean?

M1 : Perhaps, there is another interpretation for that pattern, Sir. For example, the sequence is a function.

P : What is the relationship between sequence and function?

M1 : The pattern is a function

P : What if the fourth term is not 11, just say 17?

M1 : It can be, the pattern is not arithmetic sequence

P : What sequence?

M1 : It's not arithmetic sequence. I don't know Sir.

P : So, the value of the sequence can be any number, can't it?

M1 : Yes, Sir. If it is searched, there must be a pattern

Based on interview results, M1 stated that the next term of a certain sequence can be any number with a condition that there is a pattern in that sequence. When the fourth term of sequence 2, 5, 8, ... is 11, he said that the sequence is an arithmetic sequence because it has the same pattern of difference among the term. When the fourth term is replaced with 17, he said that it is okay and the pattern is not an arithmetic sequence anymore. His statement that the pattern is in the form of function indicates that M1 has related his conception of sequence with function. This can also be shown in the following interview transcripts.

P : Why the term of the sequence are called the 1st term, the 2nd term and so on?

M1 : Because it is like counts, Sir. So, it's uses natural numbers.

P : What is the relationship of sequence term with natural number?

M1 : Because we call the first term, the second term is counts. They are elements of natural number

P : In your definition, you didn't mention natural number at all.

M1 : So, my definition is still wrong, isn't it Sir?

Basically, M1 had related the concept of sequence with the concept of function and natural number but he had not been able to state that sequence is a function of natural number to real number. He was still doubt about his conception as shown from his statement "So, my definition is still wrong, isn't it Sir?"

Fruitful

Fruitfulness status can be traced when subject was given different problems and he was required to solve them based on his conception.

P : OK. Please determine the next term of sequence of 1, 1, 2, 3, 5, 8, ...

M1 : 12 while thinking and writing

P : Why 12?

M1 : Because after the first term, 0 is added, then, 1 is added, 2 is added, 3 is added, and so on. Perhaps, any number preferred by test developer is added

P : What if the next term of sequence is 13?

M1 : It may be right, Sir

P : What do you mean?

M1 : Maybe, the pattern is by adding the first difference with the second difference

In solving another problem to determine the sequence term of a certain sequence (not arithmetic sequence), M1 tried to find the pattern first. As seen from quoted interview above, in determining the next term of sequence, M1 made a pattern by adding each term of sequence with 0, 1, 1, 2, 3, ..., so, he answered 12 as the next term of sequence 1, 1, 2, 3, 5, 8, This is because $8 + 4 = 12$. The above interview result above shows a fact that in deciding the next term of sequence M1 still considered a pattern of term difference.

3.2. Conception Status of Subject M2.

The results of subject M2's work on worksheet can be shown in Figure 2.

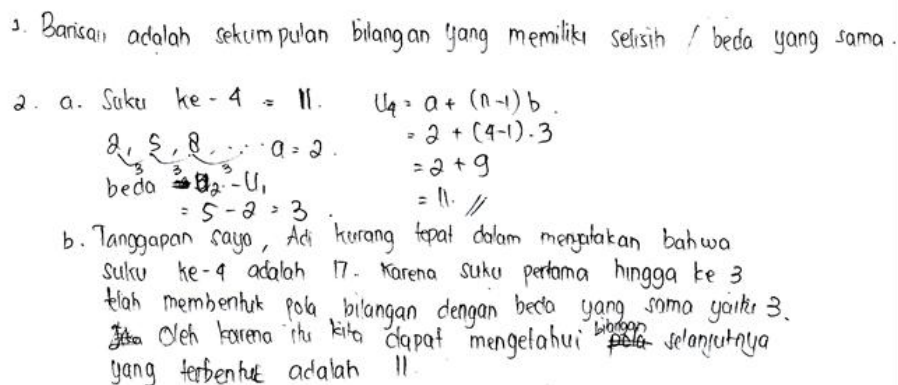


Figure 2.

The results of subject M2's work on worksheet

Based on results of written test, it was obtained M2's responses on definition of number sequence, which is a collection of numbers having the same differences. M2's conception indicates she still used arithmetic sequence term in defining sequence. Subject M2 solve question 2(a) by using arithmetic sequence formula, in that the fourth term of sequence 2, 5, 8, ... is 11. M2's responses to question 2(b) can be categorized as having disagreement with the statement that the fourth term of that sequence is 17. Her reason is that the first term to the third value had already formed a pattern with the difference of 3 among the values. Thus, it can be determine that the fourth value is 11. M2's reason was not in line with her written conception of sequence definition, where sequence is defined as a collection of numbers with the same difference. In contrast, for her reason, he used a term of pattern. The detailed conception of M2 can be explained as follows.

Intelligible

The following quoted interview transcripts shows status of M2's conception on meaning of her conception.

P : Explain your definition of sequence?

M2 : Sequence is a collection of numbers having a certain pattern

P : Why did you say different definition from what you have written before?

M2 : That's wrong, Sir. I thought (in the worksheet) it was an arithmetic sequence.

P : All right then. So, what is the key word that characterizes the sequence?

M2 : There is a pattern

P : What do you mean by pattern?

M2 : There is the formula

P : What do you mean by formula?

M2 : That sequence must have been able to be stated in its formula.

From the results of interview, there was a different conception between the result of written test and that of interview. In the written test, M2 defined sequence as a collection of numbers having the same difference. Her conception was simply based on arithmetic sequence. On the other hand, when she was interviewed, she realized her mistakes in the conception and changed it by defining that sequence is a collection of numbers having a certain pattern. What is meant by pattern in M2's conception is that every sequence has to be able to be stated in the sequence formula.

Status M2's conception in giving example and representing her conception in other forms can be shown in the following quoted interview transcripts.

P : Can you give another example of sequence?

M2 : 1, 3, 5, 7, 9, ...

P : What is the next term of sequence?

M2 : 11

P : Can you give reason that it is an example of sequence?

M2 : Because it has a pattern with difference of 2

P : Isn't it a sequence if it doesn't have a pattern?

M2 : No (with a doubt)

P : Can you give an example?

M2 : 0, 8, 5, 7, 6, 8, ... A collection of numbers but not having a pattern

P : So, what is a pattern?

M2 : The sequence can be made into formula

P : OK, 2, 5, 8, 1, 7, 9, 10, 11, ... What do you think?

M1 : That is not a sequence, there is no pattern. Except that it is 2, 5, 8, 1, 7, 9, 10, 11, ... Then the pattern is repeated again (while writing as follows)

$2, 5, 8, 1, 7, 9, 10, 11, \dots$
 $\underbrace{\quad\quad}_3 \underbrace{\quad\quad}_3 \underbrace{\quad\quad}_7 \underbrace{\quad\quad}_6 \underbrace{\quad\quad}_2 \underbrace{\quad\quad}_1 \underbrace{\quad\quad}_1 \underbrace{\quad\quad}_3 \underbrace{\quad\quad}_3 \underbrace{\quad\quad}_7$

P : So, how can you write the formula of the sequence?

M2 : How? (thinking) I'm confused

In giving another example of number sequence, subject M2 used an example of arithmetic sequence. The given example of sequence was still based on her conception that a sequence must have a certain pattern. So, when there is a collection of numbers without a pattern, it is not a sequence. As an example, she said that 0, 8, 5, 7, 6, 8, ... was not a sequence. Besides, she also stated that 2, 5, 8, 1, 7, 9, 10, 11, ... was not a sequence. If the next values form a pattern (the differences among the next values have the same pattern with those of the previous values), it will make a sequence. Furthermore, M2 said that a sequence has to be able to be made into formula. This shows that M2 has not been able to represent her conception.

Plausible

The following interview indicates how M2 determined the values of number sequence.

P : Given of sequence is 2, 5, 8, ... what is the fourth term?

M2 : I think it is 11

P : Why 11?

M2 : Because we can see the difference between 2 and 5 is 3, the difference between 5 and 8 is also 3. So, the difference with the next number is 3.

P : What if the fourth value is 17. What do you think?

M2 : I think if we use arithmetic sequence concept, it is wrong (that the fourth term is 17) because the difference is not the same with that of 2, 5, 8, which is 3.

P : Is arithmetic sequence written in that question?

M2 : No

P : So, isn't it possible if the next term is 17?

M2 : I don't know Sir

The results of interview shows that M2's conception of sequence is still limited on arithmetic sequence concept. When the value of the sequence is replaced with different value, M2 said, "I don't know". In addition, M2 simply understood that sequence concept is only related with number pattern. The following interview describes the student's understanding of the relationship of her concept of sequence with other concepts.

P : What are other concepts which are related with sequence?

M2 : Number pattern

P : Is there any other concept?

M2 : No.

Fruitful

In solving the problems of determining the value of sequence (not arithmetic sequence), M2 tried to find the formed pattern first. M2's conception in deciding sequence pattern is by firstly finding the difference of the two values in order. The following quotation of interview illustrates how M2 used her conception to solve the problem of determining the values of sequence.

P : Let 1, 1, 2, 3, 5, 8... is sequence. What is the next term of sequence?

M2 : 12

P : Why 12?

M2 : Because the pattern is 1, 1, 2, 3, 5, 8... next plus 4, so the next term of sequence is 12

P : What if the next term of sequence is 13?

M2 : Then, the difference is 5, so, the pattern is not clear

P : Isn't it a pattern?

M2 : What is the pattern, Sir?

P : The pattern is by adding the previous two values in order.

From the above interview, it is clear that if the next term of sequence 1, 2, 3, 5, 8... is 13, M2 considered that there was a pattern. The following example also indicates that M2 always considered a pattern as differences among two values in order.

P : Let 1, 5, 2, 0, 1, What is the next term of sequence?

M2 : Next term of sequence is 1.

P : Why 1?

M2 : Because the pattern is 4, 3, 2, 1, after that 0. So, the next term of sequence is 1.

The interview above indicates that M2 always tried to find pattern of difference among the values in order in deciding the next values without considering negative and positive differences.

IV. Discussion

There are many factors which trigger students' conception. Based on the findings of this study, it is clear that students' previous knowledge gotten from their previous education levels still influences their learning in higher education levels. These basic assumptions of sequence definition can inhibit the students' interpretations of their observations of the gotten information (Vosniadou, 1994). For example, students' conception that sequence is a collection of numbers having a certain pattern and must be able to be written in formula. This conception is influenced by their previous knowledge of number pattern.

Another factor causing students' conception is information from their teachers during their learning in the classroom. According to Wenning (2008), students may still hold "false ideas", that is "false or misleading statement" of teachers resulted from incorrect instructions. Teachers often have wrong conceptions of mathematic concepts. When the instruction is simply a knowledge transfer from teachers to students, there will be repeated faults of conception. Teachers' utterances and statements in the classroom will be permanently attached in students' minds, strongly believed, continuously used by students and likely resistant to changes (Treagust&Duit, 2008).

Apart from the sources of students' conception, for students, the developed conception is never wrong. The conception is said to be wrong if there is deviation or difference from scientific concepts (Subanji, 2016). Such conception can potentially distract students' learning of the subsequent concepts (Coetzee & Amanda, 2012; Taber, 2000). This can be seen from students' responses to this question "Given sequence 2, 5, 8, ... determine the fourth term of the sequence. Give your reason!" All students answered that the fourth term is 11 with a reason that the sequence is an arithmetic sequence and has the difference of 3 among the values in order. This fact may be caused by students' conception resulted from surface learning, which is learning by listening to teachers' explanation and doing exercises. In such learning, it is emphasized only on how to solve the problems without understanding the problems a whole. This, in turn, will lead to poor ability of students in understanding mathematic concepts.

Moreover, in representing the concept of sequence, students' conception is still fragmented. For example, students could not represent the general form (symbol) of sequence concept. Students' conception of sequence is fragmented on arithmetic and geometric sequences. According to diSessa (1993), sometimes, knowledge structure of students forms various conceptions on a concept, in which there is fragmentation on each other. Students' disability in representing concept of sequence is because they put emphasis on how to solve the problems without understanding the problems in great details. Such fact indirectly supports fragmented students' conception.

Status of students' conception on sequence can be classified into intelligible, not plausible and fruitful. In understanding the concept of sequence, students' conception is only focused on a collection of numbers having a pattern. Although students have stated that there is relationship between sequence and function, they could not state that sequence is a function of natural number to real number. This is in accordance with what has been indicated by Li and Tall (1992) that students faced cognitive difficulty about the idea of sequence as a function. Whereas Sfard (1991) consider that such students' conception can be categorized as an operational conception that has process, algorithm or activity meaning.

Therefore, when learning mathematic concepts, it is important for students to think that the concepts are intelligible, plausible, and fruitful so that it can encourage them to learn more meaningfully (Chen & Wang, 2016). The first step to take is to explore students' conception. If the students are aware of their conceptions, unsatisfied and unsure with their conceptions, then they try to find more intelligible, plausible, and fruitful concepts, conception change can happen.

V. Conclusion

Students' conception of sequence are likely not in accordance with scientific concepts. The status of their conception can be categorized into intelligible one, but not plausible and fruitful. The conditions of students' conception status are as follows: (1) students recognized that sequence numbers has to have a certain pattern; they could not give non-example of their conceptions; (2) students could only represent sequence with arithmetic and geometric sequences. (3) students simply related sequence with concepts of function and natural number, but had not been able to show that sequencenumbers is a function of natural number to real number. (4) in solving the problems of sequencenumbers, students' conception was only focused on sequence as differences among the terms.

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